AP Calculus AB- Semester A
Final Exam Review Sheet

Final Exam Overview

This Final Exam Review can help you prepare for the final exam by giving you an idea of what you need to study, review, and learn. To succeed, you should be thoroughly familiar with the subject matter before you attempt to take the exam. When you take the exam, please be prepared to show your competence and understanding of the Enduring Understanding and Essential Knowledge for AP Calculus (EU and EK), as determined by the College Board.

Since this Final Exam Review Sheet will not reference all the material that will be on the exam, you should use the AP Calculus Course Description to guide your exam preparation. You can view the EUs and EKs online at https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-ab-and-bc-course-and-exam-description.pdf on pages 7 through 16. You will see references to these EK numbers throughout this review sheet and in the course overviews within the course.

You must earn a score of 70% or higher on the final exam in order to pass the course. Be sure to bring your graphing calculator to the exam.

Preparing for the exam

Be aware that the Final Exam covers a full semester's worth of work; be prepared to devote an adequate amount of study and preparation time so that you are ready to take the exam. Part of your preparation for this exam depends on the work you have done throughout the course. As you worked through each unit of the course, you should have:

- read all of the information in each lesson
- viewed all videos
- responded to all interactive notebook prompts
- worked all practice problems and checked answers with keys provided
- completed all Practice Quiz questions
- completed all Graded Assignments, and
- reviewed the materials in the Resource Center.

In addition, before taking the exam, you should:

- review all the information in your journal
- review or retake the practice quizzes
- complete all sample problems in this study guide
- review lessons in the course for study guide problems you find difficult

Materials Needed

You will need to bring a #2 pencil and a graphing calculator to complete the final exam. You will receive a computer-graded answer sheet, a formula chart and scratch paper when you arrive at the testing center. *Please note: the actual AP Calculus AB examination in May will not allow a formula chart.*

Exam Structure

You will be allowed 3 hours to complete this exam. The final exam for this course will consist of 50 multiple-choice questions worth 2 points each for a total of 100 points. The exam covers a wide variety of topics. To help you study, we have divided this review sheet into two topics and provided study tips and sample questions for each.

Unit 1 Topics: Limits, Continuity and Derivatives
Unit 2 Topics: Derivatives and their applications, slope fields and related rates

Scholastic Honesty

When you arrive at the testing center you will be asked to carefully read the exam rules and sign a statement agreeing to take the exam in accordance with the rules. This is called the Examinee's Certification. The following is a copy of these rules:

Examinee's Certification

This certification must be signed *before* the exam is administered and then returned with the completed examination attached, or credit for the exam will not be given.
Scholastic dishonesty is a serious academic violation that will not be tolerated. Scholastic dishonesty encompasses, but is not limited to:

- copying from another student's work;
- using an unauthorized testing proctor or taking the exam at an unauthorized testing location;
- using materials not authorized by a testing proctor;
- possessing materials that are not authorized by a testing proctor, such as lessons, books, or notes;
- knowingly using or soliciting, in whole or Topic, the contents of an unadministered test;
- collaborating with or seeking aid from another student without authorization during the test;
- substituting for another person, or permitting another person to substitute for oneself, in taking a course test or completing any course-related assignment;
- using, buying, stealing, or transporting some or all of the contents of an unadministered test, test rubric, homework answer, or computer program.

Evidence of scholastic dishonesty will result in a grade of *F* on the examination and an *F* in the course (if applicable).

At the testing center, you will be asked to sign a statement that says you have read the above and agree to complete the examination with scholastic honesty.

Additional Study Tips

The following information provides direction for your studies. For each part, you will find study tips and sample questions to give you a general idea of the types of questions you can expect to see on the exam.

Unit 1: Limits and Continuity

This topic relates to your knowledge of finding limits for functions using graphs and algebraic techniques. In addition to limits, you should be able to justify when a function a continuous using the 3-part definition of continuity.

Study Tips for Limits and Continuity:

This topic relates to Unit 1 Lessons 1-2. Familiarize yourself with those Lesson Objectives, and then be prepared to demonstrate knowledge of the following topics:

- state the limit for a function using graphs and tables
- describe limits as x approaches infinity
- apply the 3-part definition of continuity to explain whether a function is continuous or not
- apply the intermediate value theorem and how it relates to continuity
- evaluate limits algebraically

Sample Questions for Unit 1, Lessons 1-2:

The following are sample questions. You can find the correct answers listed at the end of this review section, but try answering the questions without looking at the answers first to check your comprehension.

DIRECTIONS: Select the BEST response to each question.

1. Let f be the function defined below where b is a constant. If f is continuous at $x = 2$, what is the value of b?

$$f(x) = \begin{cases} 4 + bx^3 & \text{for } x < 2 \\ 3 + \sin(\pi x) & \text{for } x \geq 2 \end{cases}$$

A. 2

B. 1

C. 0

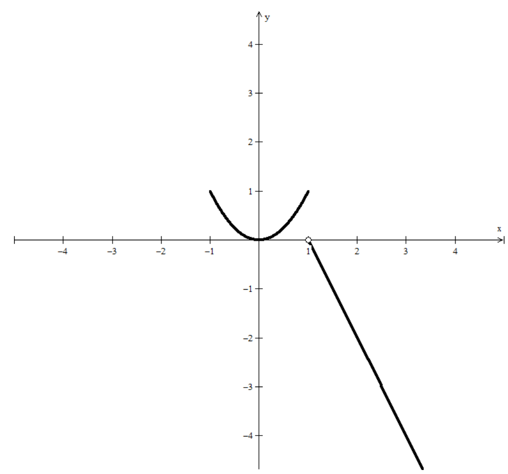
D. $-\frac{1}{8}$

2. Let f be the function defined below. Determine the following limit: $\lim_{x \to 2} f(x)$

$$f(x) = \begin{cases} x^2 + 3x & \text{for } x \le 2 \\ 4x - 2 & \text{for } x > 2 \end{cases}$$

A. nonexistent
B. 6
C. 8
D. 10

Use the graph of f(x) provided for questions #3-5.



3. Determine the $\lim_{x \to 1^+} f(x)$

A. nonexistent
B. 0
C. –1
D. 1

4. Which of the following statements must be true?

I. $f(1) = 1$

II. $\lim_{x \to 1^-} f(x) = 0$

III. $\lim_{x \to 0} f(x) = 0$

A. I only B. I and II C. I and III D. II and III

5. Which of the following explains why $f(x)$ is or is not continuous?

A. $f(x)$ is continuous because the limit exists as x approaches 1.
B. $f(x)$ is not continuous because $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$
C. $f(x)$ is continuous because is $f(c)$ defined for every value of c in the domain of [–1, 3].

D. $f(x)$ is not continuous because $f(1)$ is undefined.

6. Determine the limit: $\lim_{x\to\infty} \dfrac{x^2+6x}{5e^x+3x}$

A. 0 B. 2 C. ∞ D. $\dfrac{1}{5}$

7. Determine the value of b that would make the following statement true.

$$\lim_{x\to 4} \frac{x^2+bx-8}{x-4} = 6$$

A. 2 B. 0 C. –2 D. 1

8. Determine: $\lim_{h\to 0} \dfrac{\sin(x+h)-\sin x}{h} =$

A. 0 B. – sin x C. – cos x D. cos x

Answer Key

Item Number	Correct Answer	EK expectation
1	D	1.1A3
2	A	1.1A3
3	B	1.1B1
4	C	1.1B1, 1.1A1, 1.1A2
5	B	1.1B1, 1.1A1
6	A	1.1D2
7	C	1.1C2
8	D	2.1C1

Unit 1: Derivatives

This topic relates to your knowledge of finding derivatives and showing an understanding that a derivative represents the slope of a function.

Study Tips for Derivatives:

This topic relates to Unit 1 Lessons 2-6. Familiarize yourself with those Lesson Objectives, and then be prepared to demonstrate knowledge of the following topics:

- writing the equation of a tangent line
- using tangent line equations to find linear approximations
- using the derivative to determine the slope of a function
- finding derivatives using a variety of formulas (power, chain, product, quotient, trigonometric, transcendental)
- find derivatives for implicitly defined functions
- determine the derivative at a point using graphs, functions and tables

Sample Questions for Unit 1, Lessons 3-6:

The following are sample questions. You can find the correct answers listed at the end of this review section, but try answering the questions without looking at the answers first to check your comprehension.

1. Determine the slope of the line tangent to the graph of $y = 2\sqrt{x} - \frac{5}{x} + 3x - 2$ at $x = 1$.

A. –2 B. 9 C. 2 D. –3

Use the function $f(x) = 2x^3 - x^2 - 3x - 1$ ***for problems #2 and #3.***

2. Write the equation of a tangent line for at $x = 2$.

A. $y = (x - 5) + 2$
B. $y = 17(x - 5) + 2$
C. $y = 17(x - 2) + 5$
D. $y = 5(x - 2) + 5$

3. Use the tangent line from #2 to approximate the value of the function at $x = 2.1$.

A. –0.9 B. –47.3 C. 6.7 D. 5.5

4. The local linear approximation to the function g at $x = 2$ is $y = -2x + 3$. What is the value of $g(2) + g'(2)$?

A. –3
B. –2
C. 1
D. 5

Use the values of *f*, *f'*, *g*, and *g'* for selected values of *x* in the table provided for #5 through #8.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	0	2	1	4
-2	-2	4	-5	3
2	3	-1	6	-2
3	1	2	-1	5

5. If $h(x) = f(g(x))$, what is the value of $h'(3)$?

A. 2
B. 5
C. 7
D. 10

6. If $h(x) = f(x)g(x)$ then what is the value of $h'(2)$?

A. -12

B. $\frac{-1}{3}$

C. 0

D. 2

7. If $h(x) = \frac{g(x)}{f(x)}$, then what is the value of $h'(-2)$?

A. $\frac{-14}{25}$

B. $\frac{2}{5}$

C. $\frac{12}{25}$

D. $\frac{7}{2}$

8. If g is the inverse function of f, what is the value of $g'(1)$?

A. ½
B. 2
C. 1
D. –1

Answer Key

Item Number	Correct Answer	EK expectation
1	B	2.1C1
2	C	2.3B1
3	C	2.3B2
4	A	2.3B2
5	D	2.1B, 2.1C4
6	A	2.1B, 2.1C3
7	D	2.1B, 2.1C3
8	A	2.1B, 2.1C6

Unit Topic 2: Function Analysis using Derivatives

This topic relates to your knowledge of describing the behavior of functions using the derivative relationships for justification.

Study Tips for Function Analysis:

This topic relates to Unit 2 Lessons 1-3. Familiarize yourself with those Lesson Objectives, and then be prepared to demonstrate knowledge of the following topics:

- using the first and second derivative tests to determine relative extrema for functions
- using the sign chart and/or graph to aid in justifying conclusions about function behavior
- determine concavity using the second derivative
- connect function behaviors to the derivative in the application of motion problems
- identify critical numbers and possible points of inflection

Sample Questions for Topic 2, Lessons 1-3:

The following are sample questions. You can find the correct answers listed at the end of this review section, but try answering the questions without looking at the answers first to check your comprehension.

1. A function's derivative is given by $f'(x) = e^x (x - 3)$. For what interval(s) of x is $f(x)$ increasing?

A. $(0, 3)$
B. $(3, \infty)$
C. $(-\infty, 0)$
D. $(-\infty, 3)$

The velocity function $v(t) = (t - 3)(t - 1)^2$ describes the motion of a particle moving horizontally for $t \geq 0$. Use this information to answer questions 2 through 4.

2. When is the particle moving right?

A. $t > 3$
B. $t < 3$
C. $0 < t < 3$
D. The particle always moves left.

3. At what value(s) of time is the acceleration zero?

A. $t = 0$

B. $t = 1$

C. $t = 3$

D. $t = 1$ and $t = \frac{7}{3}$

4. Is the particle speeding up or slowing down at $t = 2$? Justify your conclusion.

A. The particle is speeding up because both $v(2)$ and $a(2)$ are negative.

B. The particle is slowing down because both $v(2)$ and $a(2)$ are negative.

C. The particle is speeding up because $v(2)$ and $a(2)$ have opposite signs.

D. The particle is slowing down because $v(2)$ and $a(2)$ have opposite signs.

A graphing calculator is required for questions 5 and 6. Analyze the function, $f(x)$, whose derivative is given by $f'(x) = e^{\cos x} + \frac{3}{x} + 2$ on the interval [–5, 7].

5. When is $f(x)$ decreasing? Justify.

A. $f(x)$ is decreasing on the interval $(0, \infty)$ because $f'(x) > 0$.
B. $f(x)$ is decreasing on the interval $(-0.731, 0)$ because $f'(x) < 0$.
C. $f(x)$ is decreasing on the interval $(0, 3.687)$ because $f''(x) < 0$.
D. $f(x)$ is decreasing on the interval $(6.255, \infty)$ because $f''(x) > 0$.

6. For what interval is $f(x)$ concave up?

A. (0, 3.687)
B. (– 0.731, 0)
C. (3.687, 6.255)
D. $(6.255, \infty)$

Answer Key

Item Number	Correct Answer	EK expectation
1	B	2.2A1
2	A	2.1C3, 2.3C1
3	D	2.1C3, 2.3C1
4	A	2.1C3, 2.3C1
5	B	2.2A1
6	C	2.2A1

Unit Topic 2: Optimization, Related Rates and Slope Fields

This topic relates to your knowledge of describing the behavior of functions using the derivative relationships for justification.

Study Tips for Derivatives:

This topic relates to Unit 2 Lessons 4-6. Familiarize yourself with those Lesson Objectives, and then be prepared to demonstrate knowledge of the following topics:

- using derivative and function relationships to find the minimum and maximum in application problems
- apply implicit differentiation in solving related rate applications
- use differential equations to generate slope fields and describe possible solution curves

Sample Questions for Topic 2, Lessons 4-6:

The following are sample questions. You can find the correct answers listed at the end of this review section, but try answering the questions without looking at the answers first to check your comprehension.

1. A girl is 12 feet from the base of a tree when she sees a squirrel in the tree. She walks towards the tree at a rate of 2 ft/sec. The squirrel is located 8 feet above the ground. By what rate is the distance between the girl and the squirrel changing?

A. $4\sqrt{3}$ ft/sec

B. $\frac{6\sqrt{13}}{13}$ft/sec

C. $\frac{6\sqrt{5}}{5}$ ft/sec

D. $\frac{8\sqrt{3}}{3}$ ft/sec

2. You have 600 feet of fencing to enclose a rectangular playing field by fencing 3 sides, since once side backs up to the community center. What dimensions will maximize the area?

A. 200 ft x 100 ft
B. 50 ft x 250 ft
C. 150 ft x 300 ft
D. 125 ft. x 125 ft

3. Given the differential equation $\frac{dy}{dx} = \frac{y(2-x)}{x}$, which of the following describes when the slope field will have horizontal tangent lines?

A. There will be horizontal tangent lines at the values of $x = 0$.

B. There will be a horizontal tangent line at the point (2,0).

C. There will be horizontal tangent lines at every point on the y-axis.

D. There will be horizontal tangent lines at the values of $x = 2$ or $y = 0$, except the point (0,0).

- *Visit https://utexas.box.com/v/SlopeFieldHorizTanLines to view an explanation of a similar problem.*

4. The function $y = e^{3x} - 2x + 1$ is a solution to which of the following differential equations? Hint: find y' and y'' to see which of the following this satisfies.

A. $y'' - 3y' - 6 = 0$
B. $y'' - y' + 6 = 0$
C. $y'' + y' + 6 = 0$
D. $y'' + 3y' - 6 = 0$

Answer Key

Item Number	Correct Answer	EK expectation
1	B	2.3C2
2	C	2.3C3
3	D	2.3F1
4	A	2.3F1

Free Response Review

This section is intended to help you view the types of free response problems you will encounter on the official AP exam in May. There are no free-response questions on the UT High School final exam, so consider these as practice only.

Study Tips for Free Response:

Be familiar with all the various theorems and formulas for use on any variety of topics. The AP grading rubric often checks for:

1) Set-up of equations
2) Derivative solution
3) Algebraic solution
4) Units, justification and explanation

Sample Questions for Free Response:

The following are sample questions. You can find the correct answers listed at the end of this review section, but try answering the questions without looking at the answers first to check your comprehension.

1. Diana rode her bicycle on a straight road. She recorded her velocity, in miles per hour, for selected values of t over the interval $0 \le t \le 2$ hours, as shown in the table.

***t* (hours)**	0	0.5	1	1.5	2
***v*(*t*) (miles per hour)**	0	10.5	14.7	16.3	13.5

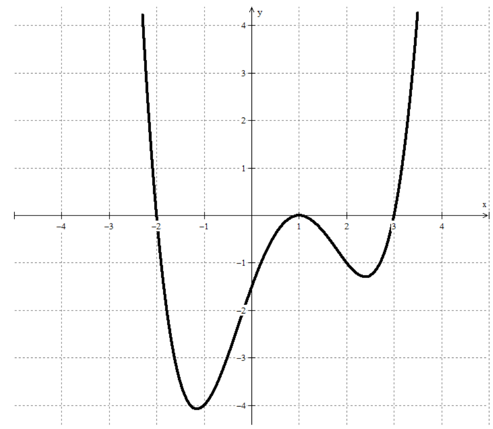
a) Use the data in the table to approximate $v'(1.25)$. Show the computations that lead to your answer. Indicate units of measure.

b) Determine whether the bicycle is speeding up or slowing down at 1.5 hours. Justify your conclusion.

c) Write a tangent line equation that can be used to approximate the velocity using the coordinate when time is 2 hours.

d) Use the tangent line equation from part c to approximate the velocity at 2.5 hours.

2. Given the graph of $f'(x)$ below, answer the following questions and give a justification for each conclusion.



a) For what values of x is $f(x)$ decreasing? Justify.

b) For what values of x is $f(x)$ concave up? Justify.

c) Identify and classify all relative extrema for $f(x)$? Justify each value.

d) Determine any points of inflection for $f(x)$. Justify.

Answer Key

#	Solution Steps and Answer	Grading Rubric
1a	$v'(1.25) = \dfrac{16.3 - 14.7}{1.5 - 1} = 3.2\ miles/hr^2$	3 points: 1 point calculations, 1 point answer, 1 point units
1b	$v(1.25) > 0$ $a(1.25) = v'(1.25) > 0$ speeding up because same signs for velocity and acceleration	3 points: 1 point calculations, 1 point answer, 1 point justification
1c	$y = v'(2)\ (x - 2) + 13.5$ $y = -5.6\ (x - 2) + 13.5$	2 points (1 point partial credit allowed)
1d	$y = -5.6\ (2.5 - 2) + 13.5 = 10.7$	2 points
2a	$(-2, 3)$ because $f'(x) < 0$. The value $x = 1$ is included. This is because the graph of $f'(x)$ does not pass through the x-axis (it bounces) which means that f(x) is decreasing at $x = 1$.	1 point answer; 1 point justification
2b	Approximately $(-1, 1)$ and $(2.5, \infty)$ because $f''(x) > 0$ (or you can say because $f'(x)$ is increasing)	1 point answer, 1 point justification
2c	$x = -2$ is a relative maximum because the derivative changes signs from positive to negative (which implies the function changes from increasing to decreasing). $x = 3$ is a relative minimum because the derivative changes signs from negative to positive (which implies the function changes from positive to negative).	2 points answer, 2 point justification
2d	About $x = -1$, 1, and 2.5 because $f''(x)$ changes signs. (or the turning points on $f'(x)$).	1 point answer, 1 point reason

Reminders: The actual AP Exam grades on a 9 point rubric.